3D FDTD Simulation of Superconductor Coplanar Waveguides

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ABSTRACT

3D superconductor coplanar waveguide structures are simulated using the 3D FDTD method with graded mesh. Using the two-fluid model, Maxwell's equations are expressed in the time domain and discretized in time and all three space directions. By using a combination of the forward and central finite difference scheme in conjunction with a graded mesh layout of second order accuracy, 3D superconductor CPW discontinuities are calculated. Tensor properties of the substrate are taken into account as well as thin buffer layers between the superconductor and the supporting substrate.

I. Introduction

Computer simulation of high-Tc superconductor microwave circuits including the tensor properties of the supporting substrate is not a trivial task [1-6]. Time domain methods seem to be attractive for this purpose because they can provide time- and frequency-domain data at the same time. The FDTD method is most notable in this respect, because of its direct approach to solve Maxwell's equation and because the method is very flexible in that it can handle arbitrarily shaped structures. Although the method has many attractive features, one commonly known disadvantage of the FDTD is that it requires large amounts of memory space and CPU time, in particular for inhomogeneous waveguide structures combining large and small subsections. Coplanar waveguides are especially difficult for the FDTD analysis. Because even for small slot dimensions, absorbing walls must be placed at a sufficient distance from the slot to avoid reflections which can deteriorate the accuracy of the results. This in turn requires discretization of a large cross-section, which may be impossible to do with most workstations.

To alleviate this problem, we have developed a variable mesh technique which provides second order accuracy and can be used to resolve the geometry of superconductor structures with a minimum of grid cells [7]. Using this variable mesh, we have investigated pulse propagation in coplanar structures [5]as well as losses and after propagation parameters in small CPW transmission lines [7,8].

For 3D superconductor coplanar structures with thin metallization and buffer layer as well as a supporting substrate with tensor properties, the FDTD method has not been successfully applied yet. Since we are only concerned with linear problems, the following 3D FDTD algorithm is derived from the two-fluid model. Instead of using the central finite difference scheme throughout the formulation, we apply the forward finite difference scheme to those terms in the field expressions representing the superconductivity, while the central finite difference scheme is applied to the remainder of the terms. This combination of two discretization schemes leads to a significantly improved stability of the overall FDTD algorithm. In conjunction with the variable mesh algorithm, it is now possible to calculate 3D discontinuities in superconductor CPW using the FDTD method without resorting to a supercomputer.

II. Theory

The two-fluid model can be used to describe the superconductivity with a complex conductivity [1-4]

$$\sigma_{sc} = \sigma_1 - j\sigma_2 \ ,$$

$$\sigma_1 = \sigma_n \theta \ ,$$

$$\sigma_2 = \frac{1}{\omega\mu(\lambda_L(T))^2},$$

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$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1-\theta}},$$

$$\theta = \left(\frac{T}{T_c}\right)^2,$$

where σ_n, $\lambda_L(0)$ and T_C are the normal conductivity near the critical temperature, zero temperature penetration depth and critical temperature of the superconductor, respectively.

For this complex conductivity, the Maxwell equation can be modified as [6,5,7]

$$\nabla \times \hat{H} = (j\omega\varepsilon + \sigma_1)\hat{E} + \frac{1}{j\omega\mu\lambda_L^2}\hat{E} \tag{1}$$

In the time domain (1) can be represented as

$$\nabla \times \hat{H} = \varepsilon\frac{\partial}{\partial t}\hat{E} + \sigma_1\hat{E} + \frac{1}{\mu\lambda_L^2}\int\hat{E}dt \tag{2}$$

Using central finite differences, the electric and magnetic fields are interleaved by half a space step. The update equation for the electric fields at time $t=(n+1)\Delta t$ at point p (i, j, k) can be obtained, where Δt is the time step, and i, j, and k are the mesh indices along the x-, y-, and z-directions. To resolve the very thin superconductor and buffer layers without using excessive computer memory space, a variable mesh layout with second order accuracy is applied [5,7]. Although this step is necessary, it is not sufficient to provide a stable 3D FDTD algorithm. The problem is that due to the high conductivity of the superconductor, very small time steps are required. To improve the stability of the algorithm is only possible by using $\sigma_1\underline{\mathbf{E}}^{(n+1)}$ instead of $\sigma_1\underline{\mathbf{E}}$ in (2). We also assume that all the fields are zero before t=0. The update equation, for example, for E_z may then be given as

$$E_z^{n+1}(i,j,k) = \frac{1}{1+\sigma_1\Delta t/\varepsilon}\left(E_z^n(i,j,k) + \frac{1}{\varepsilon_r}\left(\frac{c\Delta t}{\lambda_L}\right)^2\right.$$

$$\times\sum_{p=0}^{n}E_z^p(i,j,k) + \frac{s}{\varepsilon_r(1+\sigma_1\Delta t/\varepsilon)}$$

$$\left(\frac{H_x^{n+0.5}(i,j,k) - H_x^{n+0.5}(i,j-1,k)}{p_i}\right.$$

$$\left.-\frac{H_y^{n+0.5}(i,j,k) - H_y^{n+0.5}(i,j,k-1)}{q_j}\right), \tag{3}$$

where $s=c\Delta t/\Delta h$ with the speed of light $c=2.9979\times10^8$ m/s. Similar equations can be obtained for other field components.

To include also anisotropy of the superconducting film, the complex conductivity may be represented as a tensor

$$\bar{\sigma}_{sc} = \bar{\sigma}_1 - j\bar{\sigma}_2 \tag{4}$$

$$\bar{\sigma}_2(T) = \begin{bmatrix} \sigma_x(T) & \sigma_{xy}(T) & 0 \\ \sigma_{yx}(T) & \sigma_y(T) & 0 \\ 0 & 0 & \sigma_z(T) \end{bmatrix} \tag{5}$$

When the principle axes of the superconducting film are aligned with the coordinates, $\sigma_{yx}(T) = \sigma_{xy}(T) = 0$.

In the case of YBCO,

$$\sigma_j = \frac{1}{\omega\mu\lambda_j^2}$$

with j = x, y, z, and the penetration depths are equal along the x- and y-axes, namely, $\lambda_x = \lambda_y$. The z-axis penetration depth becomes

$$\sigma_z = \delta\sigma_{x,y} \tag{6}$$

where the coefficient δ is constant. Then the FDTD can be modified to include this tensor conductivity. The normal state conductivity along the transverse x-y plane is typically almost an order of magnitude higher than the conductivity along the z-axis.

III. Numerical results

To test the improvement of the variable mesh layout with second order accuracy over that with first order accuracy, relative error versus grading ratio and the computation time is plotted in Fig. 3. It is evident from this comparison that there is a slight increase in computation time for the second order scheme, but the improvement in accuracy is significant. The attenuation characteristics of a superconductor CPW with buffer layer are shown in Fig. 4. For the thin metal and buffer layer chosen here, the buffer layer influence is marginal. This is not the case for the propagation constant, as illustrated in Fig. 5. The tensor conductivity in this structure was chosen to be $\lambda_z = 5\lambda_{x,y}$. The typical computation time for this cross-section was about 2 hours on an IBM RS 6000 (530) workstation. The number of mesh cells was approximately 60X40X100 in x,y,z direction. Fig.6 demonstrates the application of the 3D FDTD method for the analysis of a CPW resonator. With a gap width of G= 200 µm, the resonance peak was very sharp at 15.75 GHz. The computation time for this structure was approximately 30 hours and about the same number of mesh cells as in Fig. 5 was used.

IV. Conclusion

A 3D FDTD method has been presented combining a forward and central finite difference scheme to improve the stability of the algorithm. In conjunction with a graded mesh of second order accuracy the 3D FDTD method can now be used to analyze 3D superconductor CPW discontinuities with buffer layer on workstation computers.

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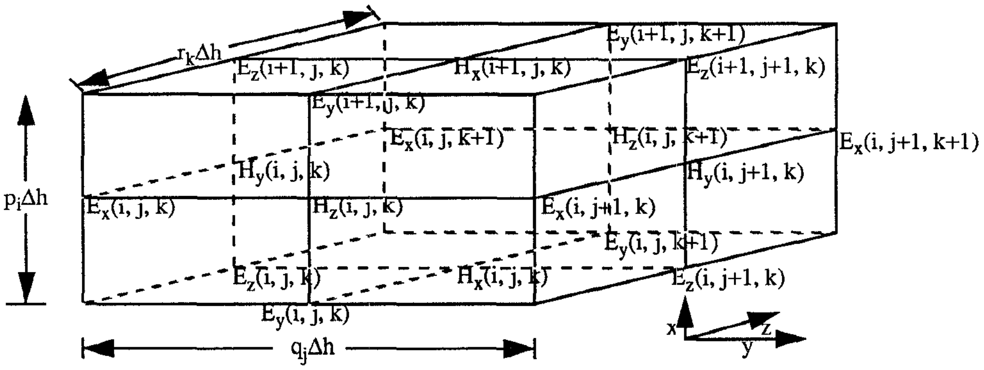


Fig. 1 Discretization unit cell

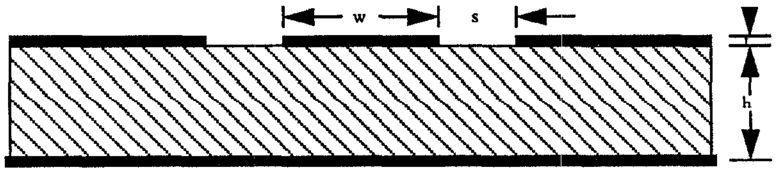


Fig. 2 Structure under simulation: Coplanar waveguide

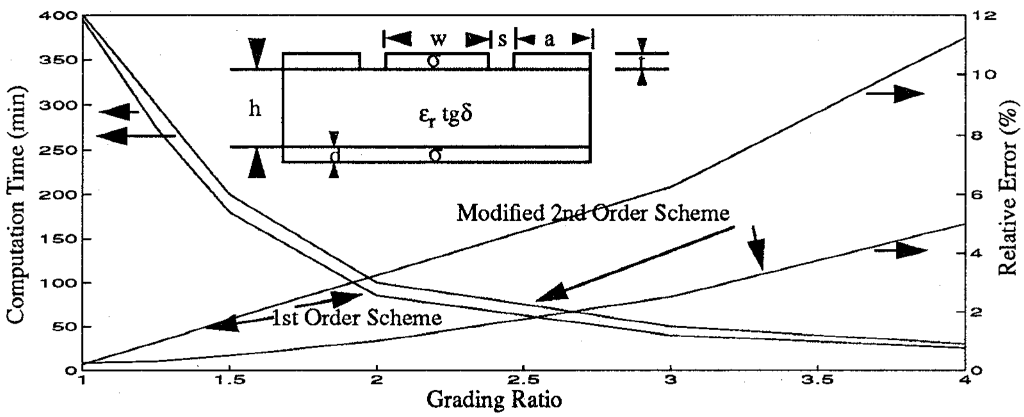


Fig. 3 Grading ratio effect on the calculation accuracy and efficiency, ε_r=12.9, tgδ=4x10^{-4}, σ=4.0x10^{7}, d=8μm, t=3μm, s=8μm, a=50μm, h=200μm

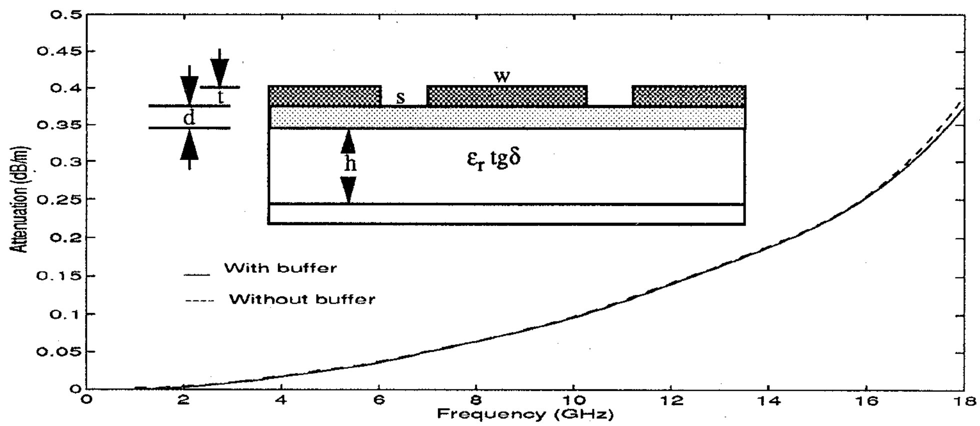
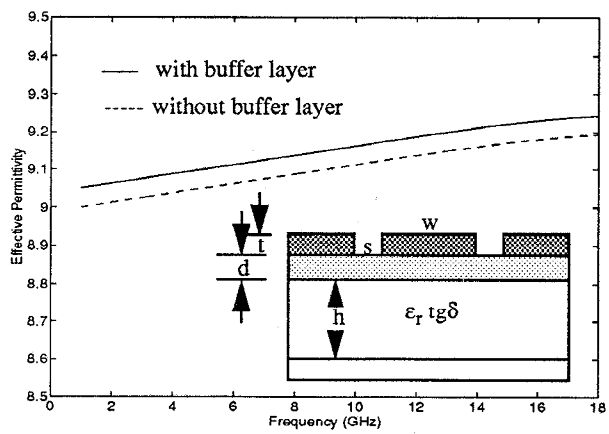


Fig. 4 Attenuation Characteristic of a superconductor CPW with and without a buffer layer(ε_r=500, d=0.1 μm), w=150 μm, s=100μm, t=0.5 μm, h=500 μm, ε_r=9.8, λ=0.2 μm, σ_1=1.0 S/μm, T_c=93 K, T=77 K, $\lambda_z=5\lambda_{x,y}$



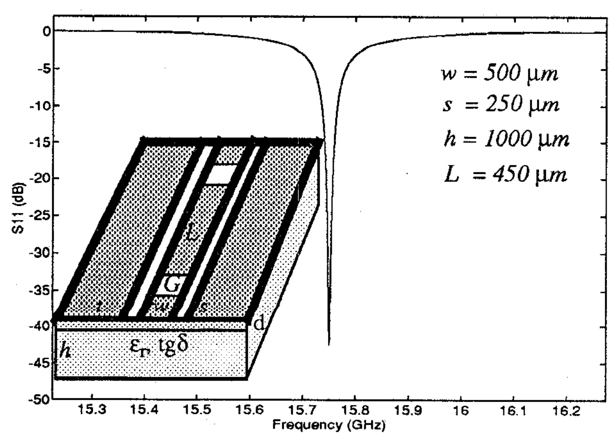


Fig. 5 Propagation Characteristic of a superconductor CPW with and without a buffer layer(ε_r=500, d=0.1 μm), w=150 μm, s=100μm, t=0.5 μm, h=500 μm, ε_r=9.8, λ=0.2 μm, σ_1=1.0 S/μm, T_c=93 K, T=77 K, $\lambda_z=5\lambda_{x,y}$

Fig. 6 CPW gap resonator, ε_r=9.8, λ=0.18 μm, σ_1=1.0 S/μm, T_c= 86 K, T=80 K $\lambda_z=5\lambda_{x,y}$, a buffer layer(ε_r=500, d=0.1 μm), t=0.5 μm